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Research Statement

My primary research interests are in the relativistic quantum theories of few-body systems and the implications of these theories for the foundations of quantum mechanics. My current focus involves applying the methods of few-body theories to understand the structure and emergent properties of relativistic resonances and unstable particles.

A quantum few-body system is understood as one that is isolated and sufficiently "simple" in the sense that it is possible to both measure a complete set of commuting observables on it and make theoretical predictions on these measurements to any desired degree of accuracy. Among the most interesting few-body systems are the particles, both elementary and composite, produced in high energy reactions. The energy scale of these reactions makes it necessary that any theory describing them must be both relativistic and quantum mechanical. In addition to quantum mechanics and relativity, few-body theories are also expected to obey the principle of cluster decomposition, a requirement that puts certain important restrictions on the form of the Hamiltonian.

The most popular framework for constructing relativistic quantum theories of few-body systems is (local) quantum field theory (QFT). While it has many attractive features and widely acknowledged successes, QFT suffers from a range of problems, both axiomatic and computational, and the prevailing opinion is that it is an approximation to a (yet to be discovered) more exact theory [1, 2]. The well-known profound mathematical difficulties such as the empty domain (in the Hilbert space) of the Hamiltonian in its standard form lead to the inference that the precise mathematical structure of QFT is not yet known. There are also formidable computational difficulties appertaining, in particular, to strongly interacting systems. Because of the infinite degrees of freedom and large coupling constants of the QFT models of such systems, it is difficult to have sufficient control over errors in any calculation that starts from the first principles. Approximate calculations that involve a finite number of renormalized Feynman diagrams –and thus suppress an infinite number of diagrams with large coupling constants– fail to assess the extent to which the tail of the perturbation series contributes to defining the form of dynamics [3]. Consequently, there exist no known algorithms for computing some of the most basic properties of strongly interacting composite systems, such as the mass of the proton.

An alternative framework for constructing theories of few-body systems entails the notion of *directly interacting particles*. In a remarkable paper published in 1949 [4], Dirac showed how the requirements of special relativity can be accommodated in a theory of directly interacting particles. The central idea is to modify the generators of Poincaré transformations by introducing interactions, which are functions of the canonical coordinates, in a manner that preserves the commutation relations of the Poincaré Lie algebra. Subsequent seminal works by Bakamjian and Thomas [5], Foldy [6], Coester [7], and Sokolov [8] extended the formalism to incorporate both quantum mechanics and cluster decomposability. Just as in QFT, Wigner's classic construction of the representations of the Poincaré group plays a fundamental role in these few-body theories. Another key feature of the relativistic quantum theories of direct-interactions is that they do not generally fulfill the micro-causality requirement of local field theories. Instead, they satisfy a weaker condition known as macro-causality that can be most easily formulated as the commutativity of the operators of space-time translations in different sub-clusters of the system in the limit of infinite spacelike separation between these sub-clusters. The direct interaction theories are the most general quantum theories that satisfy special relativity and cluster decomposability.

My recent work has been to employ the techniques of direct interaction theories to obtain a general, model independent theory of relativistic resonances and decaying states [9]. As in the Bakamjian-Thomas construction [5], this new theory exploits the possibility of introducing interactions solely into the invariant mass operator of the system of particles. When there are only two interacting particles, the simplest case, this can be quite easily done by first resolving the direct product of (non-interacting) the Poincaré group representations into a direct sum and then using the invariant mass operator on the direct sum (which has a continuous spectrum) to construct the observables for the interacting system. The interaction-incorporating observables can be so constructed as to fulfill the commutation relations of the Poincaré Lie algebra. By integrating this Lie algebra, one expects to obtain a representation of the Poincaré group that describes the interacting system.

However, it soon became evident that this expected integrability does not hold if there is any non-trivial scattering (e.g., leading to resonances) in the system. (This is reminiscent of the absence of any non-trivial local field theories that satisfy all of the Wightman axioms.) Instead, the interaction-incorporating Poincaré Lie algebra was found to integrate into representations of two subsemigroups of the Poincaré group, each consisting of Lorentz transformations and translations into either the forward or the backward light-cone. The representations of these two *causal Poincaré semigroups* are defined in two different dense subspaces of the Hilbert space, characterized by the different (in fact, complementary) analyticity properties of the in- and out- scattering states.

If scattering leads to the formation of resonances, the *S*-matrix will contain a pole in the lower half of the complex plane. The most important feature of the new theory is the existence of an *irreducible* representation of the forward Poincaré semigroup that can be uniquely characterized by the resonance pole position and the angular momentum of the relevant partial wave. In much the same way as stable particles are characterized by the irreducible representations of the Poincaré group, with the eigenvalues of their Casimir operators defining the mass and spin of the particle, resonances can be described by the irreducible representations of the causal Poincaré semigroup. The basis vectors, called Gamow vectors, for the representation space can be labeled by the eigenvalues of a complete set of commuting observables, which includes the mass operator whose eigenvalue is the resonance pole position[1]. Thus, they carry information characteristic of resonances such as the Breit-Wigner mass distribution. Furthermore, the semigroup time evolution of Gamow vectors leads to the exponential law for the decay probabilities typical of decaying states. Therefore, the Gamow vectors and the associated representations of the Poincaré semigroup provide a unifying

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[1] As an operator in the Hilbert space, the mass operator is self-adjoint. However, the theory is set up in such a way that it has an extension into a larger vector space that embeds the Hilbert space and as an operator in this larger space, the spectrum of the mass operator contains complex numbers, including the pole position.

description of resonances and decaying states, experimentally and observationally different entities that we may now call quasistable states, and also place them on an equal footing with the stable particles.

The theory of resonances and decaying states based on the Poincaré semigroup and Gamow vectors is rigorous and subsumes such widely used calculational techniques and effective theories as the Weisskopf-Wigner method and the Lee-Ohme-Yang model of the neutral Kaon, which lie *outside* of the conventional framework of quantum mechanics, as they can now be obtained as mathematically well defined approximations. The most far reaching application of the theory, however, is that it provides a fundamental principle for defining the mass, width and lifetime of quasistable states and solves the problem of gauge non-invariance of the conventional on-shell definition of the mass and width [10].

In addition to its applications in particle physics, there are important implications of the new theory for the foundations of quantum physics. As stated above, the representations of the forward and backward semigroups are defined in two dense subspaces representing the scattering out-states and in-states. Since dynamical equations of quantum mechanics are formulated as differential equations, it is necessary that the vector space of states be complete with respect to a topological structure defined on it. The conventional choice of topology is the norm topology of the Hilbert space[2], and since the spaces of in- and out- states are dense in this topology, their norm completion necessarily leads to the same Hilbert space, a property known as the asymptotic completeness. In other words, it is impossible to make manifest the operational and theoretical differences between the in- and out- scattering states if the spaces are modeled as (topologically complete) Hilbert spaces. As a result of the above study on quasistable states, it has become clear that there exit topologies in which the finer details of the in- and out- states appear in relief. It is important to emphasize that all of the fundamental axioms of quantum mechanics, including that of the scalar product structure, are consistent with these new topologies. It thus opens up the question of the minimality and uniqueness of the set of quantum mechanical axioms, an avenue of research that always interests me.

The new topologies described above can be viewed as a lifting of a degeneracy between the in- and out- scattering states, and breaking of the Poincaré symmetry leading to the semigroup representations is a direct consequence of this lifted degeneracy. In other words, it is the properties of the states of the physical system that disrupt the translational symmetry of the dynamics and lead, in particular, to an asymmetric time evolution.

My current research efforts are primarily aimed at understanding the relation of the above theory of resonance scattering and decay to QFT. My preliminary results indicate that the degeneracy between the in- and out- scattering states that is so centrally significant to the theory implies a new kind of degeneracy in the QFT vacuum state. It also appears, conversely, that it is possible to obtain both the analyticity properties of the scattering in- and out- states and the Poincaré semigroup furnished by their transformation properties from this vacuum degeneracy. Subsequently, along the lines of these analyticity properties, the appropriate propagator can be constructed to describe resonances non-perturbatively as poles

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[2] In this statement, the term “Hilbert space” does not simply mean a scalar product space. Rather, it refers to a scalar product space that is complete with respect to the ensuing norm topology.

of that propagator, thus establishing a connection between the propagator poles of QFT and the Gamow vectors of the direct interaction theory. An important component of the development that has yet to be worked out is the *CPT* extensions of the representations of the causal Poincaré semigroup. The translations defined only into the forward or backward light-cone introduce subtleties into the construction of the *CPT* operators. In fact, even the extension of the unitary representations of the Poincaré group by *CPT* shows a surprising complexity and yields a 16-fold multiplicity [11]. Some of these representations of the *CPT* extended Poincaré group appear to be promising as candidates to be adapted for the semigroup representations.

A difficult problem that requires further analysis, both within the semigroup theory itself and in its relation to QFT, is the principle of cluster decomposition. Sokolov's work [8] showed that it is possible to accommodate cluster decomposition in particle theories of direct interaction, contrary to the popular belief that fields are indispensable for fulfilling the requirement [12]. I tend to believe that in a semigroup theory, it possible to combine the irreversible evolution and cluster decomposition to a super-causality principle and thus strengthen the macro-causality property of the direct interaction theories of few-body systems. This interesting problem is among my research plans for the near future.

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